Period 1, April 16, 2025

36, 37, ~~8~~, ~~13~~, ~~16~~, ~~18~~, ~~29~~, 33, 42, ~~15~~, 38, ~~3~~, ~~14~~, ~~19~~, ~~27~~, ~~10~~, ~~11~~, ~~12~~, ~~22~~, ~~28~~, 31, 32, 35, 43, 44, 45

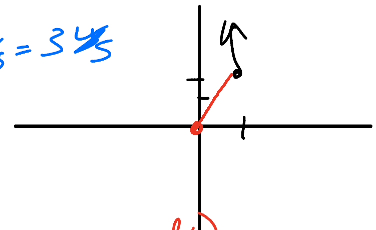
8. $y = x^2 + 1 \Rightarrow \frac{dy}{dT} = 2x\frac{dx}{dT}$

$\frac{dx}{dT} = \frac{3}{2}$ u/s

$\frac{dy}{dT} = \cancel{2}\cdot\frac{3}{\cancel{2}}$ u/s $\cdot x$

$\frac{dy}{dT} = 3 \cdot x$ u/s

(1, 2) $\frac{dy}{dT} = 3 \cdot 1$ u/s $= 3$ u/s



$$d = \sqrt{(x-0)^2 + (y-0)^2}$$

$$d = (x^2 + y^2)^{\frac{1}{2}}$$

$$\frac{dd}{dT} = \frac{1}{2}(x^2+y^2)^{\frac{1}{2}-1}\left(2x\frac{dx}{dT} + 2y\frac{dy}{dT}\right)$$

$$= \frac{1\cdot\left(2x\frac{dx}{dT} + 2y\frac{dy}{dT}\right)}{2\sqrt{x^2+y^2}} = \frac{2\cdot 1\cdot\frac{3}{2} + 2\cdot 2\cdot 3}{2\sqrt{1^2+2^2}}$$

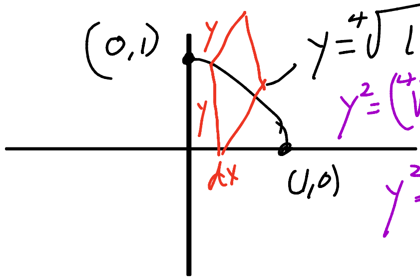
$$\frac{3+12}{2\sqrt{5}} = \frac{15\sqrt{5}}{2\sqrt{5}\sqrt{5}}$$

$$\frac{15\sqrt{5}}{10} = \frac{3\sqrt{5}}{2}$$

3. $y = \frac{e^{\ln u}}{u}$ $\qquad \frac{dy}{du} = ?$

$$\frac{dy}{du} = \frac{e^{\ln u}\cdot\frac{1}{\cancel{u}}\cdot\cancel{u} - e^{\ln u}\cdot 1}{u^2} = \frac{e^{\ln u} - e^{\ln u}}{u^2} = 0$$

10.



$y = \sqrt[4]{1-x^2}$

$y^2 = \left(\sqrt[4]{1-x^2}\right)^2$

$y^2 = \sqrt{1-x^2}$

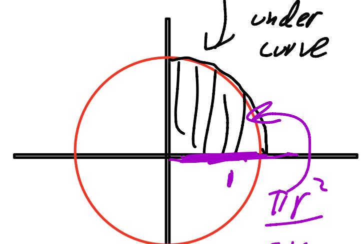
$y = \sqrt{1-x^2}$

$y^2 = 1 - x^2$

$x^2 + y^2 = 1$ Circle

$$\int_0^1 y^2\,dx = \int_0^1 \sqrt{1-x^2}\,dx$$

area under curve



$\frac{\pi r^2}{4}$

$\frac{\pi}{4} = \frac{\pi(1)^2}{4}$

11

$$\int \frac{x}{\sqrt{9-x^2}}\,dx \qquad u = 9 - x^2$$

$$du = -2x\,dx$$

$$\frac{du}{-2x} = dx$$

$$\int \frac{\cancel{x}}{\sqrt{u}} \cdot \frac{du}{-2\cancel{x}}$$

(C)

$$-\frac{1}{2}\int u^{-\frac{1}{2}}\,du$$

$$-\frac{1}{2}+1 = \frac{1}{2}$$

$$-\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{1} u^{\frac{1}{2}} + C = -\sqrt{u} + C = -\sqrt{9-x^2} + C$$

(12)

$$\int \frac{(y-1)^2}{2y}\,dy = \int \frac{y^2-2y+1}{2y}\,dy = \int\left(\frac{y^2}{2y} - \frac{2y}{2y} + \frac{1}{2y}\right)dy$$

$$\int\left(\frac{y}{2} - 1 + \frac{1}{2}\left(\frac{1}{y}\right)\right)dy \qquad (A)$$

$$\frac{1}{2}\cdot\frac{1}{2}y^{1+1=2}$$

$$\frac{y^2}{4} - y + \frac{1}{2}\ln|y| + C$$

(13)

$$\int_{\pi/6}^{\pi/2} \cot x\,dx = \int_{\pi/6}^{\pi/2} \frac{\cos x}{\sin x}\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$\frac{du}{\cos x} = dx$$

$$\int \frac{\cancel{\cos x}}{u} \cdot \frac{du}{\cancel{\cos x}} = \int \frac{du}{u} = \ln|u| + C$$

$$\ln|\sin x| + C \Big[_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$\ln\left|\sin\frac{\pi}{2}\right| - \ln\left|\sin\frac{\pi}{6}\right| = \ln 1 - \ln\left|\frac{1}{2}\right|$$

$$\ln 1 - (\ln 1 - \ln 2)$$

$$\cancel{\ln 1 - \ln 1} + \ln 2$$

$$\ln 1 - \ln\frac{1}{2} = \ln 1 - \ln 2^{-1} = \ln 1 - (-\ln 2)$$

$$= 0 + \ln 2 \qquad (C)$$

14. graph $F'(x)$
which graph is $F(x)$

$F'(2)=0$ at $F(2)$ = slope is 0
No where else

before 2 $F(x)$ increasing
after 2 decreasing

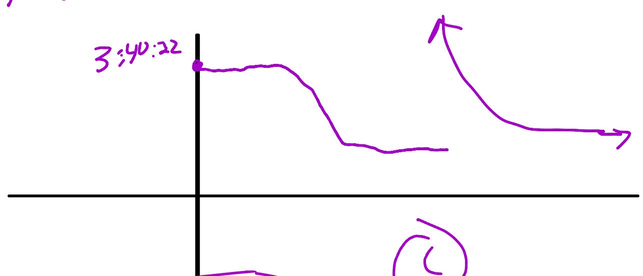
concavity concave up $(0,1)$ } = slope of $F'(x)$
concave down $(1,3)$

I and III

D

15.

3:40:22



$T > 1963$

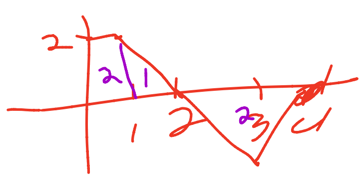
$M(T)$ = Time
must be
positive

$M'(T) = -$ because
Time goes
down

$M''(T) = +$ concave up

C

16.



$G(x) = \int_0^x F(T)\,dT$

$H(x) = \int_2^x F(T)\,dT$

$G'(x) = \cancel{H'(x+2)}$
$F(x) = \cancel{F(x+2)}$

$G(x) = \cancel{H(x+2)}$
$G(2) = \cancel{H(4)}$
$3 \cancel{= -2}$

$\boxed{G(x) = H(x) + 3}$

$\cancel{G(x) = H(x)}$
$\cancel{G(2) = H(2)}$
$3 = 0$

D

18.

as X get $\pm$ Large $\frac{dy}{dx}$ (FLatTens) $=0$

$y=\frac{2x}{x^2+1}$

as $x \to 0$ $\frac{dy}{dx}=+$

$y=\cancel{\frac{x^2}{x^2+1}}$

$\lim_{x \to \infty} \frac{x^2}{x^2+1}=1$

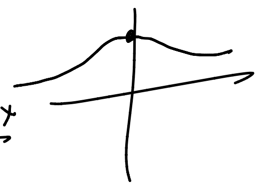
$\lim_{x \to 0^-} \frac{2x}{x^2+1}=-$

$\frac{x^2}{x^2+1} \Rightarrow \frac{dy}{dx}=\frac{2x(x^2+1)-x^2 \cdot 2x}{(x^2+1)^2}$

$\frac{\cancel{2x^3}+2x-\cancel{2x^3}}{x^2+1}$

$y=\cancel{\sin}\, x$

$y=e^{-x^2}=\frac{1}{e^{x^2}}$



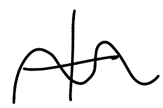
$\frac{1}{e^{x^2}}$

$-\frac{2x}{x^2+1}$

$\frac{2x}{x^2+1} \Rightarrow \frac{dy}{dx}=\frac{2(x^2+1)-2x \cdot 2x}{(x^2+1)^2}$

$2x^2+2-4x^2$

$\frac{-2x^2+2}{(x^2+1)^2}$



Sin x

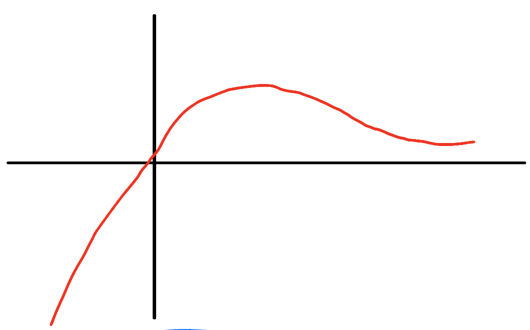
$\frac{-2x^2+2}{x^4+2x^2+1}$

$-$ when $|x|>1$

$+$ when $|x|<1$

19, 22, 27, 28, 29, 30

(19)



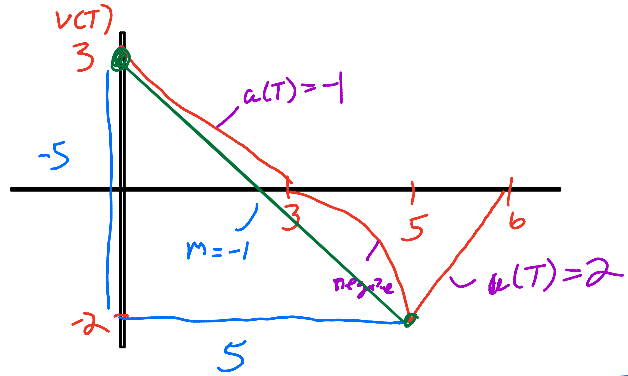
$y=\cancel{xe^x}$ as $x \to \infty$ $y \to \infty$

$\frac{x}{e^x}$ ✓

$\frac{x}{x^2+1}$ as $x \to -\infty$ $y \to 0$

$\frac{x^2}{x^3+1}$ as $x \to -\infty$ $y \to 0$

22



$a(T) = V'(T)$

slope of $V(T)$

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27. $y = \ln(4 + x^2)$

$x = +$ or $-$ x^2 is the same

I true

D

$$\frac{dy}{dx} = \frac{1}{4+x^2} \cdot 2x = \frac{2x}{4+x^2} = 0 \quad \text{when } x = 0$$

when x is $-$ $\frac{dy}{dx} = -$, when x is $+$ $\frac{dy}{dx} = +$ Min

$$\frac{d^2y}{dx^2} = \frac{2(4+x^2) - 2x(2x)}{(4+x^2)^2} = 8 + 2x^2 - 4x^2 = 8 - 2x^2 = 2(4 - x^2) = \frac{2(2-x)(2+x)}{(4+x^2)^2}$$

$(4+x^2)^2$ always $+$

$x = \pm 2$

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28. $\int_0^x f(t)\,dx = x \sin \pi x$ $F(3) =$

$F(x) = 1 \cdot \sin \pi x + x \cdot (\cos \pi x) \cdot \pi$

$F(3) = \sin 3\pi + 3 \cos 3\pi \cdot \pi$

$= 0 + 3(-1) \cdot \pi = -3\pi$

24

$$\lim_{n \to \infty} \sum_{k=1}^{n} \left(\left(\sqrt{\tfrac{2k}{n} + 3} \right) \left(\tfrac{1}{n} \right) \right)$$

interval $= 1$

$k = 1$, $n = \infty$ $\sqrt{3}$

$k = n$, $n \to \infty$ $\sqrt{5}$

(a) $\int_3^4 \sqrt{2x}\,dx$ $\sqrt{6}$ and $\sqrt{8}$

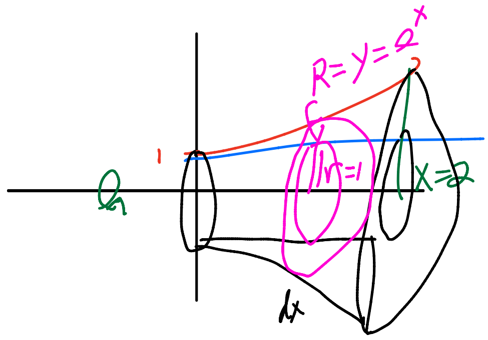
(b) $\int_0^1 \sqrt{2x+3}$ $\sqrt{3}$ $\sqrt{5}$

(c) $\int_0^1 \sqrt{2x}$ $\sqrt{0}$ $\sqrt{2}$

(d) ${}_3$ $\sqrt{2x+3}$ $\sqrt{9}$ $\sqrt{11}$

30 $y=e^x$ $y=1$ $x=2$

washer $\pi\int(R^2-r^2)\,dx$



$\pi\int((e^x)^2-1^2)\,dx$

$\pi\int(e^{2x}-1)\,dx$

C

31

$V=12\sqrt{s}$

s = distance from origin

$s=1$ when $T=0$

$\frac{ds}{dT}=12\sqrt{s}$

$\frac{ds}{\sqrt{s}}=\frac{12\sqrt{s}\cdot dT}{\sqrt{s}}$

$\int s^{-\frac{1}{2}}\,ds=\int 12\,dT$

$2s^{\frac{1}{2}}=12T+C$

$2(1)^{\frac{1}{2}}=12(0)+C$

$2=C$

$2s^{\frac{1}{2}}=12T+2$

$T=1$

$2\sqrt{s}=12(1)+2$

$2\sqrt{s}=14$

$\sqrt{s}=7$

$s=49$

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32 $F(x)=x^2-4x-5$ $x=K$ area A and B are equal

$=(x-5)(x+1)$

$\int_{-1}^{5}(x^2-4x-5)\,dx=-36$

$\int_{5}^{K}(x^2-4x-5)\,dx=36$

$\frac{1}{3}x^3-2x^2-5x\Big|_5^K=36$

$36=\frac{K^3}{3}-2K^2-5K-\left(\frac{125}{3}-50-25\right)$

$K=8$

(33)

$$\frac{dB}{dT} = \frac{k}{T}$$

$$dB = \frac{k}{T} dT$$

$$B = k \cdot \ln T + C$$

$$200 = k + C$$

200 Triples in 10 hours

$$P = P_0 e^{kT}$$

$T=0 \quad P=200 \quad P_0=200$

$T=10 \quad P=600$

$$600 = 200e^{10k}$$

$$3 = e^{10k}$$

$$\ln 3 = 10k$$

$$\frac{\ln 3}{10} = k = 0.1098$$

$$P = 200e^{0.1098T}$$

$$2793 \approx 200e^{0.1098(24)}$$